I. TRACKING A ROBOT ARM

This example illustrates the power of the Cubature Kalman Filter/Smoother (CKF/CKS). Fig. 1 shows two typical positions of a two-link robot arm, namely, ‘elbow-up’ and ‘elbow-down’. Given the angles \((\alpha_1, \alpha_2)\), the end effector position of the robot arm can be described in the Cartesian coordinate as follows:

\[
\begin{align*}
y_1 &= r_1 \cos(\alpha_1) - r_2 \cos(\alpha_1 + \alpha_2) \\
y_2 &= r_1 \sin(\alpha_1) - r_2 \sin(\alpha_1 + \alpha_2),
\end{align*}
\]

where \(r_1 = 0.8\) and \(r_2 = 0.2\) are the lengths of the two links; \(\alpha_1 \in [0.3, 1.2]\) and \(\alpha_2 \in [\pi/2, 3\pi/2]\) are the joint angles confined to a specific region. The mapping from \((\alpha_1, \alpha_2)\) to \((y_1, y_2)\) is called the forward kinematic, whereas the inverse kinematic refers to the mapping from \((y_1, y_2)\) to \((\alpha_1, \alpha_2)\). The inverse kinematic is not a one-to-one mapping and thus its solution is not unique.

For the inverse kinematic problem, let the state vector \(\mathbf{x}\) be \(\mathbf{x} = [\alpha_1 \alpha_2]^T\) and the measurement vector \(\mathbf{y}\) be \(\mathbf{y} = [y_1 y_2]^T\). The state-space model of the problem is written as

**State equation:**  \(\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{w}_k\)

**Measurement equation:**  \(\mathbf{y}_k = \begin{pmatrix} \cos(\alpha_{1,k}) & -\cos(\alpha_{1,k} + \alpha_{2,k}) \\ \sin(\alpha_{1,k}) & -\sin(\alpha_{1,k} + \alpha_{2,k}) \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} + \mathbf{v}_k\)

Here, we assume the state equation to follow a random-walk model perturbed by white Gaussian noise.
Fig. 2. Tracking results (True trajectory- Solid line, Estimated Trajectory- Dotted line)

(a) Cubature Kalman Filter (CKF)  
(b) Cubature Kalman Smoother (CKS)

Fig. 3. Ensemble averaged (over 50 runs) root mean-squared error (RMSE) results (true rmse- red line, estimated rmse- blue)

(w ∼ N(0, diag[0.01, 0.1])). The measurement equation is nonlinear with additive measurement noise v ∼ N(0, 0.005I), where I is the two-dimensional identity matrix. As can be seen from Fig. 2, α₁ is a slowly increasing process with periodic random walk whereas α₂ is a periodic, fast, and linearly-increasing/decreasing process. As depicted by Figs. 3(a) and 3(b), the root mean squared error of the CKS is less than that of the CKF as expected.

PS: Please find more about Cubature Filtering at http://grads.ece.mcmaster.ca/ aienkaran/.